

Quark and gluon TMD correlators in momentum and coordinate space

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Transverse momentum dependent (TMD) distribution correlators can be parametrized in terms of TMD parton distribution functions (PDFs), or TMDs for short. We provide an overview of the leading-twist quark and gluon TMDs, both in momentum space and in coordinate space (also called b_T -space, where b_T is Fourier conjugate to the partonic momentum k_T). We consider unpolarized, vector polarized, as well as tensor polarized hadrons, which is relevant for spin-0, spin-1/2, and spin-1 hadrons.

I. INTRODUCTION

The extraction of a parton from a hadron is mathematically described by a distribution correlator. A correlator cannot be calculated using perturbative quantum chromodynamics (QCD). Rather, it is parametrized in terms of parton distribution functions (PDFs). The so-called collinear PDFs are probability densities in the longitudinal momentum fraction x of the parton with respect to the parent hadron. Going beyond a collinear treatment, one can define transverse momentum dependent (TMD) PDFs, or TMDs for short, that besides x also depend on the partonic transverse momentum k_T . The TMDs encode the three-dimensional inner structure of hadrons. Employing the lightlike vectors $\bar{n}^\mu \equiv [1, 0, \mathbf{0}_T]$ and $n^\mu \equiv [0, 1, \mathbf{0}_T]$, the hadron and parton momenta P and k are given by:¹

$$P^\mu = P^+ \bar{n}^\mu + \frac{M^2}{2P^+} n^\mu, \quad (1)$$

$$k^\mu = xP^+ \bar{n}^\mu + k^- n^\mu + k_T^\mu, \quad (2)$$

where M is the mass of the hadron.

The inclusion of hadron polarization forms a key component of TMD phenomenology. An ensemble of spin-1/2 hadrons can be unpolarized or vector polarized. To describe the degree of vector polarization, we make use of a spin vector S . For spin-1 hadrons we need besides S also a symmetric traceless spin tensor T to describe tensor polarized states. Satisfying $P \cdot S = 0$ and $P_\mu T^{\mu\nu} = 0$, the spin vector and tensor can be parametrized as follows [1]:

$$S^\mu = S_L \frac{P^+}{M} \bar{n}^\mu - S_L \frac{M}{2P^+} n^\mu + S_T^\mu, \quad (3)$$

$$T^{\mu\nu} = \frac{1}{2} \left[\frac{4}{3} S_{LL} \frac{(P^+)^2}{M^2} \bar{n}^\mu \bar{n}^\nu + \frac{P^+}{M} \bar{n}^{\{\mu} S_{LT}^{\nu\}} - \frac{2}{3} S_{LL} \left(\bar{n}^{\{\mu} n^{\nu\}} - g_T^{\mu\nu} \right) + S_{TT}^{\mu\nu} - \frac{M}{2P^+} n^{\{\mu} S_{LT}^{\nu\}} + \frac{1}{3} S_{LL} \frac{M^2}{(P^+)^2} n^\mu n^\nu \right], \quad (4)$$

where curly brackets denote symmetrization of indices, and $g_T^{\mu\nu} \equiv g^{\mu\nu} - \bar{n}^{\{\mu} n^{\nu\}}$ with nonzero components $g_T^{11} = g_T^{22} = -1$. The spin vector has three independent parameters, namely S_L and the two transverse components of S_T , whereas the spin tensor has five, namely S_{LL} , the two transverse components of S_{LT} , as well as the two independent components of the symmetric traceless transverse tensor S_{TT} .

Parametrizations of hadronic correlators in terms of TMDs are usually given in momentum space (or k_T -space). However, for certain applications such as the implementation of TMD evolution, correlators are studied in coordinate

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¹ We define the light-cone components of a four-vector a in terms of its Minkowski components as $a^\pm \equiv (a^0 \pm a^3)/\sqrt{2}$ (the transverse components are unchanged). Using light-cone coordinates, we can represent a four-vector as $a^\mu = [a^+, a^-, \mathbf{a}_T]$. We also define $a_T^\mu \equiv [0, 0, \mathbf{a}_T]$, so that $a_T^2 = -\mathbf{a}_T^2$.

space (or b_T -space, where b_T is Fourier conjugate to k_T). To ensure a one-to-one correspondence between TMDs in k_T -space and b_T -space, it is essential that a correlator is parametrized in terms of TMDs of definite rank. To this end, a correlator in k_T -space needs to be parametrized using symmetric traceless tensors in k_T [2]. Up to rank $n = 4$, the symmetric traceless tensors $k_T^{i_1 \dots i_n}$ are given by

$$k_T^{ij} \equiv k_T^i k_T^j + \frac{1}{2} \mathbf{k}_T^2 g_T^{ij}, \quad (5)$$

$$k_T^{ijk} \equiv k_T^i k_T^j k_T^k + \frac{1}{4} \mathbf{k}_T^2 (g_T^{ij} k_T^k + g_T^{ik} k_T^j + g_T^{jk} k_T^i), \quad (6)$$

$$k_T^{ijkl} \equiv k_T^i k_T^j k_T^k k_T^l + \frac{1}{6} \mathbf{k}_T^2 (g_T^{ij} k_T^{kl} + g_T^{ik} k_T^{jl} + g_T^{il} k_T^{jk} + g_T^{jk} k_T^{il} + g_T^{jl} k_T^{ik} + g_T^{kl} k_T^{ij}) - \frac{1}{8} \mathbf{k}_T^4 (g_T^{ij} g_T^{kl} + g_T^{ik} g_T^{jl} + g_T^{il} g_T^{jk}), \quad (7)$$

satisfying

$$g_{Tij} k_T^{ij} = g_{Tij} k_T^{ijk} = g_{Tij} k_T^{ijkl} = 0. \quad (8)$$

See appendix C of ref. [2] for useful identities involving products of these tensors. The symmetric traceless tensor $k_T^{i_1 \dots i_n}$ of rank $n \geq 1$ only has two independent components, which allows for the following decomposition:

$$k_T^{i_1 \dots i_n} \rightarrow \frac{|\mathbf{k}_T|^n}{2^{n-1}} e^{\pm i n \varphi}, \quad (9)$$

in terms of the two real numbers $|\mathbf{k}_T|$ and φ , the polar coordinates of the transverse vector k_T .

II. CORRELATORS IN MOMENTUM SPACE

A. The quark-quark TMD correlator

The quark TMDs that appear in the parametrization of the quark-quark correlator were introduced in the nineties in refs. [3, 4] for unpolarized and vector polarized hadrons, and in 2000 also tensor polarized hadrons were considered [1]. For the quark TMDs the notation of ref. [1] is used here, which also coincides with the notation in refs. [3, 4] for the unpolarized and vector polarized cases. The quark-quark TMD correlator for spin-1 hadrons is given by²

$$\Phi_{ij}(x, \mathbf{k}_T) \equiv \int \frac{d\xi^- d^2 \xi_T}{(2\pi)^3} e^{ik \cdot \xi} \langle P, S, T | \bar{\psi}_j(0) U_{[0, \xi]} \psi_i(\xi) | P, S, T \rangle \Big|_{\xi^+ = 0}, \quad (10)$$

where a summation over color is implicitly assumed. The gauge link $U_{[0, \xi]}$ is needed for color gauge invariance and gives rise to a process dependence of the TMDs.

The correlator in eq. (10) can be parametrized in terms of TMDs. To establish a one-to-one correspondence between the TMDs in momentum and coordinate space, we employ symmetric traceless tensors built from k_T (see section I). This was already done in ref. [2] for the case of gluons, and in this proceedings contribution the same is done for the quark case as well. Separating the various possible hadronic polarization states, the correlator in eq. (10) can be parametrized in terms of leading-twist quark TMDs of definite rank as follows:

$$\Phi(x, \mathbf{k}_T) = \Phi_U(x, \mathbf{k}_T) + \Phi_L(x, \mathbf{k}_T) + \Phi_T(x, \mathbf{k}_T) + \Phi_{LL}(x, \mathbf{k}_T) + \Phi_{LT}(x, \mathbf{k}_T) + \Phi_{TT}(x, \mathbf{k}_T), \quad (11)$$

where³

$$\Phi_U(x, \mathbf{k}_T) = \frac{1}{2} \left[\not{n} f_1(x, \mathbf{k}_T^2) + \frac{\sigma_{\mu\nu} k_T^\mu \bar{n}^\nu}{M} h_1^\perp(x, \mathbf{k}_T^2) \right], \quad (12)$$

$$\Phi_L(x, \mathbf{k}_T) = \frac{1}{2} \left[\gamma^5 \not{n} S_L g_1(x, \mathbf{k}_T^2) + \frac{i \sigma_{\mu\nu} \gamma^5 \bar{n}^\mu k_T^\nu S_L}{M} h_{1L}^\perp(x, \mathbf{k}_T^2) \right], \quad (13)$$

² To obtain the correlator for a spin-0 or spin-1/2 hadron, one simply sets $S = T = 0$ or $T = 0$ respectively.

³ To avoid clutter, we suppress in this proceedings contribution in the names of the functions a reference to quarks, gluons, or gauge links.

$$\begin{aligned} \Phi_T(x, \mathbf{k}_T) = \frac{1}{2} \left[\frac{\not{k}_T \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, \mathbf{k}_T^2) + \frac{\gamma^5 \not{k}_T \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{k}_T^2) \right. \\ \left. + i\sigma_{\mu\nu} \gamma^5 \bar{n}^\mu S_T^\nu h_1(x, \mathbf{k}_T^2) - \frac{i\sigma_{\mu\nu} \gamma^5 \bar{n}^\mu k_T^{\nu\rho} S_{T\rho}}{M^2} h_{1T}^\perp(x, \mathbf{k}_T^2) \right], \end{aligned} \quad (14)$$

$$\Phi_{LL}(x, \mathbf{k}_T) = \frac{1}{2} \left[\not{k}_T S_{LL} f_{1LL}(x, \mathbf{k}_T^2) + \frac{\sigma_{\mu\nu} k_T^\mu \bar{n}^\nu S_{LL}}{M} h_{1LL}^\perp(x, \mathbf{k}_T^2) \right], \quad (15)$$

$$\begin{aligned} \Phi_{LT}(x, \mathbf{k}_T) = \frac{1}{2} \left[\frac{\not{k}_T \cdot \mathbf{S}_{LT}}{M} f_{1LT}(x, \mathbf{k}_T^2) + \frac{\gamma^5 \not{k}_T \epsilon_T^{S_{LT} k_T}}{M} g_{1LT}(x, \mathbf{k}_T^2) \right. \\ \left. + \sigma_{\mu\nu} \bar{n}^\nu S_{LT}^\mu h_{1LT}(x, \mathbf{k}_T^2) - \frac{\sigma_{\mu\nu} \bar{n}^\nu k_T^{\mu\rho} S_{LT\rho}}{M^2} h_{1LT}^\perp(x, \mathbf{k}_T^2) \right], \end{aligned} \quad (16)$$

$$\begin{aligned} \Phi_{TT}(x, \mathbf{k}_T) = \frac{1}{2} \left[\frac{\not{k}_T k_T^{\mu\nu} S_{TT\mu\nu}}{M^2} f_{1TT}(x, \mathbf{k}_T^2) - \frac{\gamma^5 \not{k}_T \epsilon_{T\mu\nu} k_T^{\mu\rho} S_{TT\rho}^\nu}{M^2} g_{1TT}(x, \mathbf{k}_T^2) \right. \\ \left. - \frac{\sigma_{\mu\nu} \bar{n}^\nu k_T^{\mu\rho} S_{TT\rho}^\mu}{M} h_{1TT}(x, \mathbf{k}_T^2) + \frac{\sigma_{\mu\nu} \bar{n}^\nu k_T^{\mu\rho\sigma} S_{TT\rho\sigma}}{M^3} h_{1TT}^\perp(x, \mathbf{k}_T^2) \right], \end{aligned} \quad (17)$$

where we have employed the notation $\epsilon_T^{ab} \equiv \epsilon_T^{\mu\nu} a_\mu b_\nu$, with $\epsilon_T^{\mu\nu} \equiv \epsilon^{\mu\nu-+}$ (the nonzero components are $\epsilon_T^{12} = -\epsilon_T^{21} = 1$).

The functions h_{1T} , h'_{1LT} , and h'_{1TT} that appear in the original parametrizations in refs. [1, 3] have been replaced by⁴

$$h_1(x, \mathbf{k}_T^2) \equiv h_{1T}(x, \mathbf{k}_T^2) + \frac{\mathbf{k}_T^2}{2M^2} h_{1T}^\perp(x, \mathbf{k}_T^2), \quad (18)$$

$$h_{1LT}(x, \mathbf{k}_T^2) \equiv h'_{1LT}(x, \mathbf{k}_T^2) + \frac{\mathbf{k}_T^2}{2M^2} h_{1LT}^\perp(x, \mathbf{k}_T^2), \quad (19)$$

$$h_{1TT}(x, \mathbf{k}_T^2) \equiv h'_{1TT}(x, \mathbf{k}_T^2) + \frac{\mathbf{k}_T^2}{2M^2} h_{1TT}^\perp(x, \mathbf{k}_T^2). \quad (20)$$

Furthermore, we have defined $g_1 \equiv g_{1L}$.

B. The gluon-gluon TMD correlator

The gluon-gluon TMD correlator can be parametrized in terms of gluon TMDs. They were first introduced for unpolarized and vector polarized hadrons in 2001 in ref. [5]. A new nomenclature was proposed a couple of years later in ref. [6]. This year, also the case of tensor polarized hadrons has been considered [2]. The gluon-gluon TMD correlator for spin-1 hadrons is given by

$$\Gamma^{\mu\nu;\rho\sigma}(x, \mathbf{k}_T) \equiv \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3} e^{ik \cdot \xi} \langle P, S, T | F^{\mu\nu}(0) U_{[0,\xi]} F^{\rho\sigma}(\xi) U'_{[\xi,0]} | P, S, T \rangle \Big|_{\xi^+=0}, \quad (21)$$

where a trace in color space is implicitly assumed. Two process-dependent gauge links $U_{[0,\xi]}$ and $U'_{[\xi,0]}$ are needed for color gauge invariance.

At leading twist, the gluon-gluon TMD correlator is given by $\Gamma^{ij}(x, \mathbf{k}_T) \equiv \Gamma^{+i;+j}(x, \mathbf{k}_T)$, where i, j are transverse indices. Separating the various possible hadronic polarization states, this correlator can be parametrized in terms of leading-twist gluon TMDs of definite rank as follows [2]:

$$\Gamma^{ij}(x, \mathbf{k}_T) = \Gamma_U^{ij}(x, \mathbf{k}_T) + \Gamma_L^{ij}(x, \mathbf{k}_T) + \Gamma_T^{ij}(x, \mathbf{k}_T) + \Gamma_{LL}^{ij}(x, \mathbf{k}_T) + \Gamma_{LT}^{ij}(x, \mathbf{k}_T) + \Gamma_{TT}^{ij}(x, \mathbf{k}_T), \quad (22)$$

where

$$\Gamma_U^{ij}(x, \mathbf{k}_T) = \frac{xP^+}{2} \left[-g_T^{ij} f_1(x, \mathbf{k}_T^2) + \frac{k_T^{ij}}{M^2} h_1^\perp(x, \mathbf{k}_T^2) \right], \quad (23)$$

⁴ These definitions were already proposed in ref. [1], so there is no conflict of notation.

$$\Gamma_L^{ij}(x, \mathbf{k}_T) = \frac{xP^+}{2} \left[i\epsilon_T^{ij} S_L g_1(x, \mathbf{k}_T^2) + \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_L}{2M^2} h_{1L}^\perp(x, \mathbf{k}_T^2) \right], \quad (24)$$

$$\Gamma_T^{ij}(x, \mathbf{k}_T) = \frac{xP^+}{2} \left[-\frac{g_T^{ij} \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, \mathbf{k}_T^2) + \frac{i\epsilon_T^{ij} \mathbf{k}_T \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{k}_T^2) \right. \\ \left. - \frac{\epsilon_T^{k_T \{i} S_T^{j\}} + \epsilon_T^{S_T \{i} k_T^{j\}}}{4M} h_1(x, \mathbf{k}_T^2) - \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_T}{2M^3} h_{1T}^\perp(x, \mathbf{k}_T^2) \right], \quad (25)$$

$$\Gamma_{LL}^{ij}(x, \mathbf{k}_T) = \frac{xP^+}{2} \left[-g_T^{ij} S_{LL} f_{1LL}(x, \mathbf{k}_T^2) + \frac{k_T^{ij} S_{LL}}{M^2} h_{1LL}^\perp(x, \mathbf{k}_T^2) \right], \quad (26)$$

$$\Gamma_{LT}^{ij}(x, \mathbf{k}_T) = \frac{xP^+}{2} \left[-\frac{g_T^{ij} \mathbf{k}_T \cdot \mathbf{S}_{LT}}{M} f_{1LT}(x, \mathbf{k}_T^2) + \frac{i\epsilon_T^{ij} \epsilon_T^{S_{LT} k_T}}{M} g_{1LT}(x, \mathbf{k}_T^2) \right. \\ \left. + \frac{S_{LT}^{\{i} k_T^{j\}}}{M} h_{1LT}(x, \mathbf{k}_T^2) + \frac{k_T^{ij\alpha} S_{LT\alpha}}{M^3} h_{1LT}^\perp(x, \mathbf{k}_T^2) \right], \quad (27)$$

$$\Gamma_{TT}^{ij}(x, \mathbf{k}_T) = \frac{xP^+}{2} \left[-\frac{g_T^{ij} k_T^{\alpha\beta} S_{TT\alpha\beta}}{M^2} f_{1TT}(x, \mathbf{k}_T^2) + \frac{i\epsilon_T^{ij} \epsilon_T^{\beta\gamma} k_T^{\gamma\alpha} S_{TT\alpha\beta}}{M^2} g_{1TT}(x, \mathbf{k}_T^2) \right. \\ \left. + S_{TT}^{ij} h_{1TT}(x, \mathbf{k}_T^2) + \frac{S_{TT\alpha}^{\{i} k_T^{j\}\alpha}}{M^2} h_{1TT}^\perp(x, \mathbf{k}_T^2) + \frac{k_T^{ij\alpha\beta} S_{TT\alpha\beta}}{M^4} h_{1TT}^{\perp\perp}(x, \mathbf{k}_T^2) \right]. \quad (28)$$

With respect to the original nomenclature in ref. [6], we have defined $g_1 \equiv g_{1L}$, as well as

$$h_1(x, \mathbf{k}_T^2) \equiv h_{1T}(x, \mathbf{k}_T^2) + \frac{\mathbf{k}_T^2}{2M^2} h_{1T}^\perp(x, \mathbf{k}_T^2), \quad (29)$$

as is explained in detail in ref. [2].

III. CORRELATORS IN COORDINATE SPACE

The correlators that we have introduced in the previous section can be translated to b_T -space, where b_T is Fourier conjugate to \mathbf{k}_T . Let us denote a generic TMD correlator by $\chi(x, \mathbf{k}_T)$ and a generic TMD function by $f(x, \mathbf{k}_T^2)$. Their counterparts in b_T -space are related by a Fourier transformation [2, 7]:

$$\tilde{\chi}(x, \mathbf{b}_T) \equiv \int d^2 \mathbf{k}_T e^{i\mathbf{k}_T \cdot \mathbf{b}_T} \chi(x, \mathbf{k}_T), \quad (30)$$

$$\tilde{f}(x, \mathbf{b}_T) \equiv \int d^2 \mathbf{k}_T e^{i\mathbf{k}_T \cdot \mathbf{b}_T} f(x, \mathbf{k}_T^2). \quad (31)$$

In momentum space, the parametrization of the correlator in terms of TMDs f_j with rank $n = n(j) \geq 0$, takes the form⁵

$$\chi(x, \mathbf{k}_T) = \sum_j C_j \frac{k_T^{i_1 \dots i_n}}{M^n} f_j(x, \mathbf{k}_T^2), \quad (32)$$

where C_j is a coefficient independent of k that contains information on both the hadron and parton polarization. Using eqs. (9), (30), and (32), the correlator in b_T -space is given by

$$\tilde{\chi}(x, \mathbf{b}_T) = \sum_j C_j \int d^2 \mathbf{k}_T e^{i\mathbf{k}_T \cdot \mathbf{b}_T} \frac{k_T^{i_1 \dots i_n}}{M^n} f_j(x, \mathbf{k}_T^2)$$

⁵ For simplicity, possible Lorentz indices on χ or C_j are omitted.

$$\begin{aligned}
&= \sum_j C_j \frac{b_T^{i_1 \dots i_n}}{M^n} \int_0^\infty d|\mathbf{k}_T| |\mathbf{k}_T| \left(\frac{|\mathbf{k}_T|}{|\mathbf{b}_T|} \right)^n (2\pi i^n) J_n(|\mathbf{k}_T| |\mathbf{b}_T|) f_j(x, \mathbf{k}_T^2) \\
&= \sum_j \frac{i^n}{n!} C_j M^n b_T^{i_1 \dots i_n} \tilde{f}_j^{(n)}(x, \mathbf{b}_T^2),
\end{aligned} \tag{33}$$

where on the second line the Bessel function of the first kind $J_k(z)$ arose from the integral identity

$$\int_0^{2\pi} d\alpha e^{ik\alpha} e^{iz \cos(\alpha-\beta)} = 2\pi i^k J_k(z) e^{ik\beta}. \tag{34}$$

Following the conventions in refs. [2, 7], we define the function $\tilde{f}^{(n)}$ as

$$\begin{aligned}
\tilde{f}^{(n)}(x, \mathbf{b}_T^2) &\equiv n! \left(-\frac{2}{M^2} \frac{\partial}{\partial \mathbf{b}_T^2} \right)^n \tilde{f}(x, \mathbf{b}_T^2) \\
&= \frac{2\pi n!}{M^{2n}} \int_0^\infty d|\mathbf{k}_T| |\mathbf{k}_T| \left(\frac{|\mathbf{k}_T|}{|\mathbf{b}_T|} \right)^n J_n(|\mathbf{k}_T| |\mathbf{b}_T|) f(x, \mathbf{k}_T^2),
\end{aligned} \tag{35}$$

where we used eq. (31) as well as the recurrence relation

$$\left(\frac{1}{z} \frac{d}{dz} \right)^m \left[\frac{J_k(z)}{z^k} \right] = (-1)^m \frac{J_{k+m}(z)}{z^{k+m}}, \tag{36}$$

with $z = |\mathbf{k}_T| |\mathbf{b}_T|$ (with $|\mathbf{k}_T|$ fixed), $m = n$, and $k = 0$.

From eqs. (33) and (35) it follows that in the parametrization of the correlator in b_T -space the n th derivative $\tilde{f}^{(n)}$ with respect to \mathbf{b}_T^2 appears rather than the function $\tilde{f} = \tilde{f}^{(0)}$ itself. Furthermore, we infer from eq. (35) that for definite-rank TMDs there is a one-to-one correspondence between the functions in momentum and coordinate space. The motivation for this particular definition of $\tilde{f}^{(n)}$ in eq. (35) becomes obvious once we set $\mathbf{b}_T = 0$, which is equivalent to integration over transverse momentum in k_T -space. Using the limit

$$\lim_{z \rightarrow 0} \frac{J_k(z)}{z^k} = \frac{1}{2^k k!}, \tag{37}$$

we see that

$$\lim_{|\mathbf{b}_T| \rightarrow 0} \tilde{f}^{(n)}(x, \mathbf{b}_T^2) = \int d^2 \mathbf{k}_T \left(\frac{\mathbf{k}_T^2}{2M^2} \right)^n f(x, \mathbf{k}_T^2), \tag{38}$$

which is precisely the conventional n th moment $f^{(n)}(x)$ of the TMD. Hence, by construction the derivatives in b_T -space are directly related to moments in k_T -space.

A. The quark-quark TMD correlator

We can use eq. (33) to translate the quark-quark TMD correlator in eq. (11) to b_T -space. In ref. [7] this was already done for spin-1/2 hadrons. For the spin-1 case we have

$$\tilde{\Phi}(x, \mathbf{b}_T) = \tilde{\Phi}_U(x, \mathbf{b}_T) + \tilde{\Phi}_L(x, \mathbf{b}_T) + \tilde{\Phi}_T(x, \mathbf{b}_T) + \tilde{\Phi}_{LL}(x, \mathbf{b}_T) + \tilde{\Phi}_{LT}(x, \mathbf{b}_T) + \tilde{\Phi}_{TT}(x, \mathbf{b}_T), \tag{39}$$

where

$$\tilde{\Phi}_U(x, \mathbf{b}_T) = \frac{1}{2} \left[\not{\mathbf{b}} \tilde{f}_1(x, \mathbf{b}_T^2) + iM \sigma_{\mu\nu} b_T^\mu \bar{n}^\nu \tilde{h}_1^{\perp(1)}(x, \mathbf{b}_T^2) \right], \tag{40}$$

$$\tilde{\Phi}_L(x, \mathbf{b}_T) = \frac{1}{2} \left[\gamma^5 \not{\mathbf{b}} S_L \tilde{g}_1(x, \mathbf{b}_T^2) - M \sigma_{\mu\nu} \gamma^5 \bar{n}^\mu b_T^\nu S_L \tilde{h}_{1L}^{\perp(1)}(x, \mathbf{b}_T^2) \right], \tag{41}$$

$$\tilde{\Phi}_T(x, \mathbf{b}_T) = \frac{1}{2} \left[iM \not{\mathbf{b}} \epsilon_T^{S_T b_T} \tilde{f}_{1T}^{\perp(1)}(x, \mathbf{b}_T^2) + iM \gamma^5 \not{\mathbf{b}} \cdot \mathbf{S}_T \tilde{g}_{1T}^{(1)}(x, \mathbf{b}_T^2) \right]$$

$$+ i\sigma_{\mu\nu}\gamma^5\bar{n}^\mu S_T^\nu \tilde{h}_1(x, \mathbf{b}_T^2) + \frac{iM^2\sigma_{\mu\nu}\gamma^5\bar{n}^\mu b_T^{\nu\rho} S_{T\rho}}{2} \tilde{h}_{1T}^{\perp(2)}(x, \mathbf{b}_T^2) \Big], \quad (42)$$

$$\tilde{\Phi}_{LL}(x, \mathbf{b}_T) = \frac{1}{2} \left[\not{n} S_{LL} \tilde{f}_{1LL}(x, \mathbf{b}_T^2) + iM\sigma_{\mu\nu} b_T^\mu \bar{n}^\nu S_{LL} \tilde{h}_{1LL}^{\perp(1)}(x, \mathbf{b}_T^2) \right], \quad (43)$$

$$\begin{aligned} \tilde{\Phi}_{LT}(x, \mathbf{b}_T) = \frac{1}{2} \Big[& iM\not{n} \mathbf{b}_T \cdot \mathbf{S}_{LT} \tilde{f}_{1LT}^{(1)}(x, \mathbf{b}_T^2) + iM\gamma^5 \not{n} \epsilon_T^{S_{LT} b_T} \tilde{g}_{1LT}^{(1)}(x, \mathbf{b}_T^2) \\ & + \sigma_{\mu\nu} \bar{n}^\nu S_{LT}^\mu \tilde{h}_{1LT}(x, \mathbf{b}_T^2) + \frac{M^2\sigma_{\mu\nu} \bar{n}^\nu b_T^{\mu\rho} S_{LT\rho}}{2} \tilde{h}_{1LT}^{\perp(2)}(x, \mathbf{b}_T^2) \Big], \end{aligned} \quad (44)$$

$$\begin{aligned} \tilde{\Phi}_{TT}(x, \mathbf{b}_T) = \frac{1}{2} \Big[& -\frac{M^2\not{n} b_T^{\mu\nu} S_{TT\mu\nu}}{2} \tilde{f}_{1TT}^{(2)}(x, \mathbf{b}_T^2) + \frac{M^2\gamma^5 \not{n} \epsilon_{T\mu\nu} b_T^{\mu\rho} S_{TT\rho}^\nu}{2} \tilde{g}_{1TT}^{(2)}(x, \mathbf{b}_T^2) \\ & - iM\sigma_{\mu\nu} \bar{n}^\nu b_T^\rho S_{TT\rho}^\mu \tilde{h}_{1TT}^{(1)}(x, \mathbf{b}_T^2) - \frac{iM^3\sigma_{\mu\nu} \bar{n}^\nu b_T^{\mu\rho\sigma} S_{TT\rho\sigma}}{6} \tilde{h}_{1TT}^{\perp(3)}(x, \mathbf{b}_T^2) \Big]. \end{aligned} \quad (45)$$

The quark TMDs in b_T -space are one-to-one related to their k_T -space counterparts through eq. (35).

B. The gluon-gluon TMD correlator

We can use eq. (33) to translate the gluon-gluon TMD correlator in eq. (22) to b_T -space. It is given by [2]

$$\tilde{\Gamma}^{ij}(x, \mathbf{b}_T) = \tilde{\Gamma}_U^{ij}(x, \mathbf{b}_T) + \tilde{\Gamma}_L^{ij}(x, \mathbf{b}_T) + \tilde{\Gamma}_T^{ij}(x, \mathbf{b}_T) + \tilde{\Gamma}_{LL}^{ij}(x, \mathbf{b}_T) + \tilde{\Gamma}_{LT}^{ij}(x, \mathbf{b}_T) + \tilde{\Gamma}_{TT}^{ij}(x, \mathbf{b}_T), \quad (46)$$

where

$$\tilde{\Gamma}_U^{ij}(x, \mathbf{b}_T) = \frac{xP^+}{2} \left[-g_T^{ij} \tilde{f}_1(x, \mathbf{b}_T^2) - \frac{M^2 b_T^{ij}}{2} \tilde{h}_1^{\perp(2)}(x, \mathbf{b}_T^2) \right], \quad (47)$$

$$\tilde{\Gamma}_L^{ij}(x, \mathbf{b}_T) = \frac{xP^+}{2} \left[i\epsilon_T^{ij} S_L \tilde{g}_1(x, \mathbf{b}_T^2) - \frac{M^2 \epsilon_T^{\{i} b_T^{j\}\alpha} S_L}{4} \tilde{h}_{1L}^{\perp(2)}(x, \mathbf{b}_T^2) \right], \quad (48)$$

$$\begin{aligned} \tilde{\Gamma}_T^{ij}(x, \mathbf{b}_T) = \frac{xP^+}{2} \Big[& -iMg_T^{ij} \epsilon_T^{S_T b_T} \tilde{f}_{1T}^{\perp(1)}(x, \mathbf{b}_T^2) - M\epsilon_T^{ij} \mathbf{b}_T \cdot \mathbf{S}_T \tilde{g}_{1T}^{(1)}(x, \mathbf{b}_T^2) \\ & - \frac{iM(\epsilon_T^{b_T\{i} S_T^{j\}} + \epsilon_T^{S_T\{i} b_T^{j\}})}{4} \tilde{h}_1^{(1)}(x, \mathbf{b}_T^2) + \frac{iM^3 \epsilon_T^{\{i} b_T^{j\}\alpha} S_T}{12} \tilde{h}_{1T}^{\perp(3)}(x, \mathbf{b}_T^2) \Big], \end{aligned} \quad (49)$$

$$\tilde{\Gamma}_{LL}^{ij}(x, \mathbf{b}_T) = \frac{xP^+}{2} \left[-g_T^{ij} S_{LL} \tilde{f}_{1LL}(x, \mathbf{b}_T^2) - \frac{M^2 b_T^{ij} S_{LL}}{2} \tilde{h}_{1LL}^{\perp(2)}(x, \mathbf{b}_T^2) \right], \quad (50)$$

$$\begin{aligned} \tilde{\Gamma}_{LT}^{ij}(x, \mathbf{b}_T) = \frac{xP^+}{2} \Big[& -iMg_T^{ij} \mathbf{b}_T \cdot \mathbf{S}_{LT} \tilde{f}_{1LT}^{(1)}(x, \mathbf{b}_T^2) - M\epsilon_T^{ij} \epsilon_T^{S_{LT} b_T} \tilde{g}_{1LT}^{(1)}(x, \mathbf{b}_T^2) \\ & + iMS_{LT}^{\{i} b_T^{j\}} \tilde{h}_{1LT}^{(1)}(x, \mathbf{b}_T^2) - \frac{iM^3 b_T^{ij\alpha} S_{LT\alpha}}{6} \tilde{h}_{1LT}^{\perp(3)}(x, \mathbf{b}_T^2) \Big], \end{aligned} \quad (51)$$

$$\begin{aligned} \tilde{\Gamma}_{TT}^{ij}(x, \mathbf{b}_T) = \frac{xP^+}{2} \Big[& \frac{M^2 g_T^{ij} b_T^{\alpha\beta} S_{TT\alpha\beta}}{2} \tilde{f}_{1TT}^{(2)}(x, \mathbf{b}_T^2) - \frac{iM^2 \epsilon_T^{ij} \epsilon_T^{\beta} b_T^{\gamma\alpha} S_{TT\alpha\beta}}{2} \tilde{g}_{1TT}^{(2)}(x, \mathbf{b}_T^2) \\ & + S_{TT}^{ij} \tilde{h}_{1TT}(x, \mathbf{b}_T^2) - \frac{M^2 S_{TT\alpha}^{\{i} b_T^{j\}\alpha}}{2} \tilde{h}_{1TT}^{\perp(2)}(x, \mathbf{b}_T^2) + \frac{M^4 b_T^{ij\alpha\beta} S_{TT\alpha\beta}}{24} \tilde{h}_{1TT}^{\perp\perp(4)}(x, \mathbf{b}_T^2) \Big]. \end{aligned} \quad (52)$$

The gluon TMDs in b_T -space are one-to-one related to their k_T -space counterparts through eq. (35).

IV. OVERVIEW

In this section we give a brief overview of all leading-twist TMDs that appear in the study of spin-0, spin-1/2, and spin-1 hadrons. In table I we have organized the quark and gluon TMDs from eqs. (11) and (22) by hadron and parton polarizations. Of the eighteen different quark TMDs, nine are odd under time reversal (T), whereas in the gluon case six out of nineteen functions are T -odd.

Quarks	γ^+	$\gamma^+\gamma^5$	$i\sigma^{i+}\gamma^5$
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp
LL	f_{1LL}		h_{1LL}^\perp
LT	f_{1LT}	g_{1LT}	h_{1LT}, h_{1LT}^\perp
TT	f_{1TT}	g_{1TT}	h_{1TT}, h_{1TT}^\perp

Gluons	$-g_T^{ij}$	$i\epsilon_T^{ij}$	$k_T^i, k_T^{ij}, \text{etc.}$
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp
LL	f_{1LL}		h_{1LL}^\perp
LT	f_{1LT}	g_{1LT}	h_{1LT}, h_{1LT}^\perp
TT	f_{1TT}	g_{1TT}	$h_{1TT}, h_{1TT}^\perp, h_{1TT}^{\perp\perp}$

TABLE I. An overview of the leading-twist quark and gluon TMDs for unpolarized (U), vector polarized (L or T), and tensor polarized (LL, LT, or TT) hadrons. The functions indicated in boldface also occur as collinear PDFs, and the ones in red are T -odd. The Dirac structures γ^+ , $\gamma^+\gamma^5$, and $i\sigma^{i+}\gamma^5 = \frac{1}{2}[\gamma^+, \gamma^i]\gamma^5$ correspond to unpolarized, longitudinally polarized, and transversely polarized quarks respectively, whereas the Lorentz structures $-g_T^{ij}$, $i\epsilon_T^{ij}$, and $k_T^i, k_T^{ij}, \text{etc.}$ correspond to unpolarized, circularly polarized, and linearly polarized gluons respectively.

An additional benefit of using definite-rank TMDs is that the functions also appearing in the collinear case (i.e. those that survive integration over k_T) are simply the rank-0 functions. However, for the quark TMDs there is one exception to this, namely the rank-0 Bacchetta function h_{1LT} . Since this function is T -odd, it cannot exist in the collinear case. Even though it survives integration over k_T , it in fact vanishes due to the combined effect of hermiticity and time reversal constraints [8]. From table I it follows that for both quarks and gluons there are four TMDs that also have a collinear counterpart.

V. CONCLUSION

The parametrizations of TMD correlators in terms of TMDs are usually studied in momentum space. However, certain QCD techniques are applied to coordinate rather than momentum space, such as the implementation of TMD evolution. Hence, it is important to have a one-to-one correspondence between TMDs in both spaces. This is achieved by ensuring that TMDs are of definite rank, which is accomplished by using symmetric traceless tensors in k_T in the parametrizations. The TMDs in b_T -space are then related to their counterparts in k_T -space through a transformation involving a Bessel function of which the order is equal to the rank of the TMD. In this proceedings contribution we have provided the leading-twist parametrizations, in both momentum and coordinate space, of the quark-quark and gluon-gluon TMD distribution correlators in terms of quark and gluon TMDs. We have considered hadrons that unpolarized, vector polarized, or tensor polarized, which is relevant for experiments involving spin-0, spin-1/2, or spin-1 hadrons.

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